THE CURVE OF THE NORTH STYLOBATE OF THE PARTHENON

Professor Constantine Caratheodory published an article concerning this curve in the Ἀρχαιολογικὴ Ἐφημερίς, 1937, p. 120. The article gives the reader the impression that the curve was intended to be the arc of a circle of great radius (R = 5560 m., that is, more than $5\frac{1}{2}$ kilometers). The Professor is of the opinion that many properties of the circle, the simplest of all curves, were known at the time the Parthenon was built, but that it was not until the middle of the 4th century B.C. that the mathematician Menaichmos discovered the parabola, ellipse, and hyperbola. Hence, the Professor argues, the curve of the stylobate of the Parthenon was a circle. It hardly seems possible, however, that the conic sections, which possess delightful intricacies, sprang fully formed from the brain of any one man, like fully armed Athena from the head of Zeus. Without doubt he codified and amplified such treatises on the conics as had been written before his day.

A good many articles have been written on the curve of the north stylobate of the Parthenon. As Professor Caratheodory's article is the most recent and is based upon careful measurements taken recently by Nicholas Balanos, the Professor's article deserves thorough study.

All four stylobates of the Parthenon—north, south, east, and west—are cut to curves. The north stylobate is the best preserved, and, on that account, writers have confined their attention chiefly to the curve of this stylobate.

The following dates are of interest for our discussion:

447–432 B.C. The Parthenon was in the process of building. The upper courses of the foundation of the Older Parthenon are built to a graceful curve, indicating that here was the case of a crowned stylobate before the time of the Parthenon.

First century B.C., latter part. In Vitruvius' treatise on architecture appears the following (Vitr., III, iv, 5): "The level of the stylobate must be increased along the middle by the *scamilli impares*: for, if the stylobate is laid perfectly level, it will look to the eye as though it were hollowed a little. At the end of my book a figure will be found, with a description, showing how the *scamilli* may be made to fit this purpose." Unfortunately the "figure" and "description" have not come down to us.

1837. The English architect John Pennethorne was the first to discover the curve of the stylobate of the Parthenon. This occurred soon after the mediaeval structures built in and around the Parthenon were removed.

1851. Penrose published careful measurements and studies of the curves (*Principles of Athenian Architecture*). He believed the curves of the stylobates approached more nearly parabolas than any other curves. He gave no proof of his belief, however.

1934. G. P. Stevens showed how the curve of the north stylobate of the Parthenon might have been laid out by means of the *scamilli impares* (*A.J.A.*, XXXVIII, 1934, pp. 533-542 and pl. XXXVII). The method produces a parabolic curve.

1936. Nicholas Balanos remeasured the curves, but he did not publish his measurements until 1940 (Ἡ Ἀναστήλωσις τῶν Μνημείων τῆς Ἀκροπόλεως; his preface is dated 1936). Balanos does not discuss the nature of the curves. His measurements agree closely with those of Penrose.

1937. C. Caratheodory, using Balanos' measurements, published the article already referred to. He investigated the curve from a mathematical point of view.

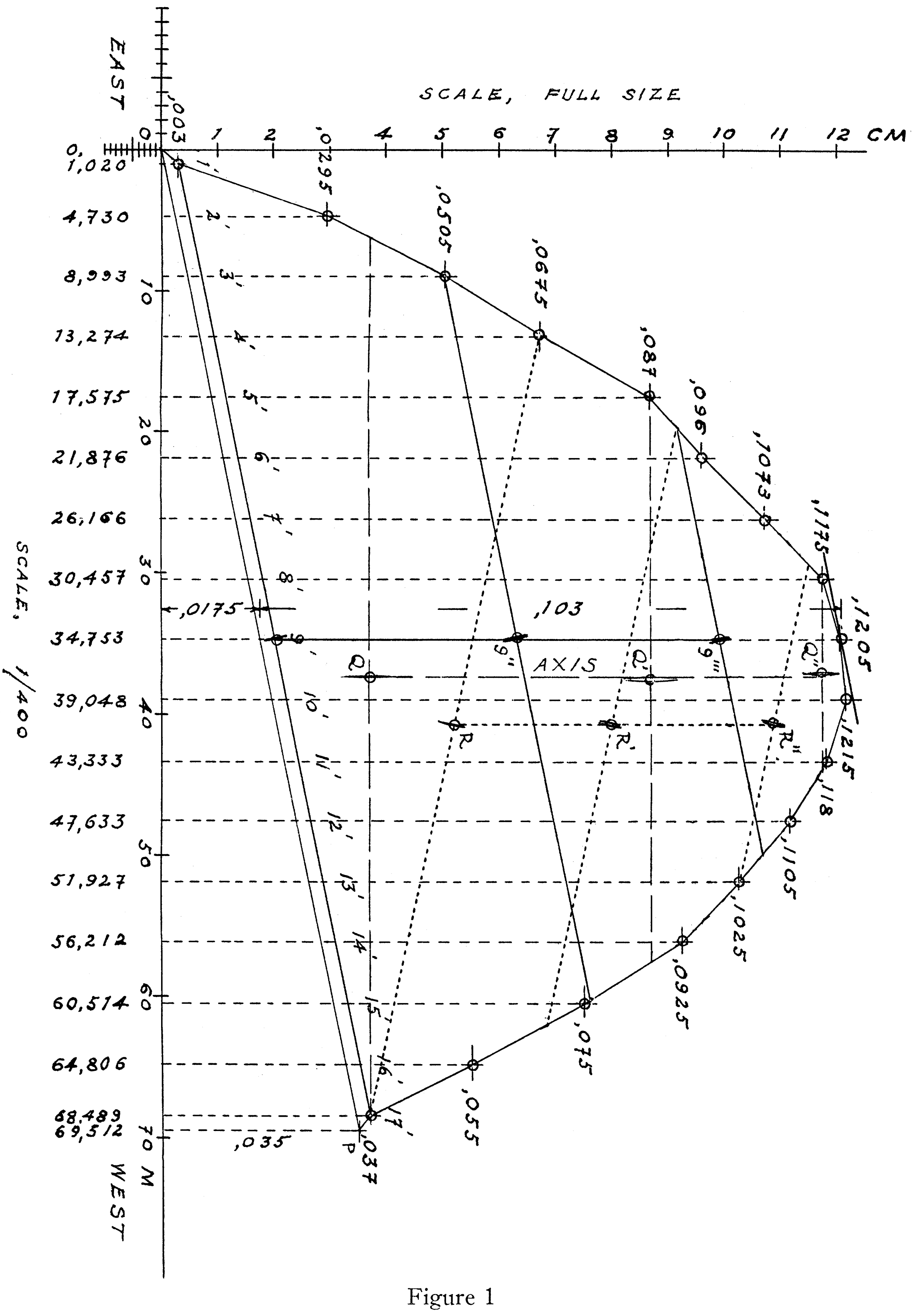
The writer's arguments for the belief that Ictinus, the architect of the Parthenon, employed a parabola, are as follows:

Using Balanos' figures, plot the horizontal measurements at a scale of 1:400, a convenient scale for our purpose (cf. Fig. 1). 1′, 2′, 3′ . . . 17′ are the axes of the 17 columns along the north flank of the temple. Next plot the vertical measurements at full size. Join the tops of the vertical measurements, forming the broken curve shown in the drawing (points O and P represent the extremities of the curve of the stylobate, P being 0.035 m. above O).

We note that the curve between 1′ and 17′ [1] is not quite a perfect curve. The irregularity is due to the following causes:

1. Without doubt Ictinus made a drawing for the curve (cf. *A.J.A.*, XXXVIII, 1934, Pl. XXXVII). Even a good drawing can only approach mathematical accuracy.
2. In the process of laying out the long and delicate curve on the stylobate there were opportunities for errors to creep in.
3. When the curve was cut in marble, there were possibilities for further variations from the architect's intention.
4. Earthquakes have disturbed the curve to a certain degree.

[1] We deal with 1′ and 17′, not with O and P. This is because the corners O and P are in such a poor state of preservation that their levels cannot be calculated with precision. Corner O is badly worn; corner P is completely gone. On the other hand, points 1′ and 17′ are well preserved, as they lie in the hollow of flutes.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AXIS
Q
Q'
Q''
R
R'
R''
P

Figure 1

5. The explosion of 1687, when many of the columns of the flanks of the temple were blown over, has contributed to the unevenness of the curve.
6. The engineers whom Balanos asked to measure the curve can only have made close approximations in their readings.

That the curve shown in the drawing is as regular as it is, is a high testimonial to the solid manner in which the foundations and steps of the Parthenon were built, to the precision with which the curve of the stylobate was cut, and to the careful readings taken by Balanos' engineers.

The curve in the drawing (Fig. 1) almost perfectly represents a parabola. The proof depends upon the following well-known theorem: *the loci of the center points of all systems of parallel chords of a parabola are straight lines parallel to the axis of the parabola.* In Figure 1 there are three systems of parallel chords. They are those which have their center points at 9′, 9″, 9‴ and Q Q′ Q″ and R R′ R″. It should be noted that the lines 9′ 9″ 9‴, Q Q′ Q″ and R R′ R″ are straight lines parallel to each other—the requirement for a parabolic curve. Further, the loci are vertical lines—this means that the axis of the parabola is vertical, and that that particular axis which is perpendicular to its corresponding chords is the axis of the parabola itself.

Objection may be raised that there are an infinite number of systems of parallel chords while only three systems have been used in the demonstration. In reply, it may be observed that the three systems were selected so that the chords of these systems would cut the curve in points—15 in number—located at fairly regular distances along the curve. The 15 points furnish sufficient data to establish the nature of the curve. There can be little doubt that the curve of Figure 1 is a very close approximation to a parabola, in fact so close, that for our purposes we may call it a parabola.

If we multiply the horizontal distances of Figure 1 by any number, the resulting curve will be a parabola with its axis parallel to that of the first parabola (cf. *A.J.A.*, XXXVIII, 1934, figs. 4 to 7 incl. and accompanying text, pp. 537-539). If we multiply by 400, we obtain the curve of the stylobate itself. Therefore, if the curve represented in Figure 1 is a parabola with a vertical axis, the curve of the stylobate was likewise a parabola with a vertical axis. Further, if the extremities of the stylobate had been on the same level, the axis of the parabola of Figure 2 would be at the center of the stylobate and there the axis would remain when the horizontal distances are multiplied by 400.

The northwest corner of the stylobate is 0.035 m. higher than the northeast corner (cf. Fig. 1). This difference in level does not seem to be intentional. The corners are 69.512 m. apart, and the slight difference in level between them cannot be remarked by the human eye. We may claim, then, that Ictinus, in his drawing of the curve, represented the corners as being on the same level (cf. Fig. 2). The axis of the

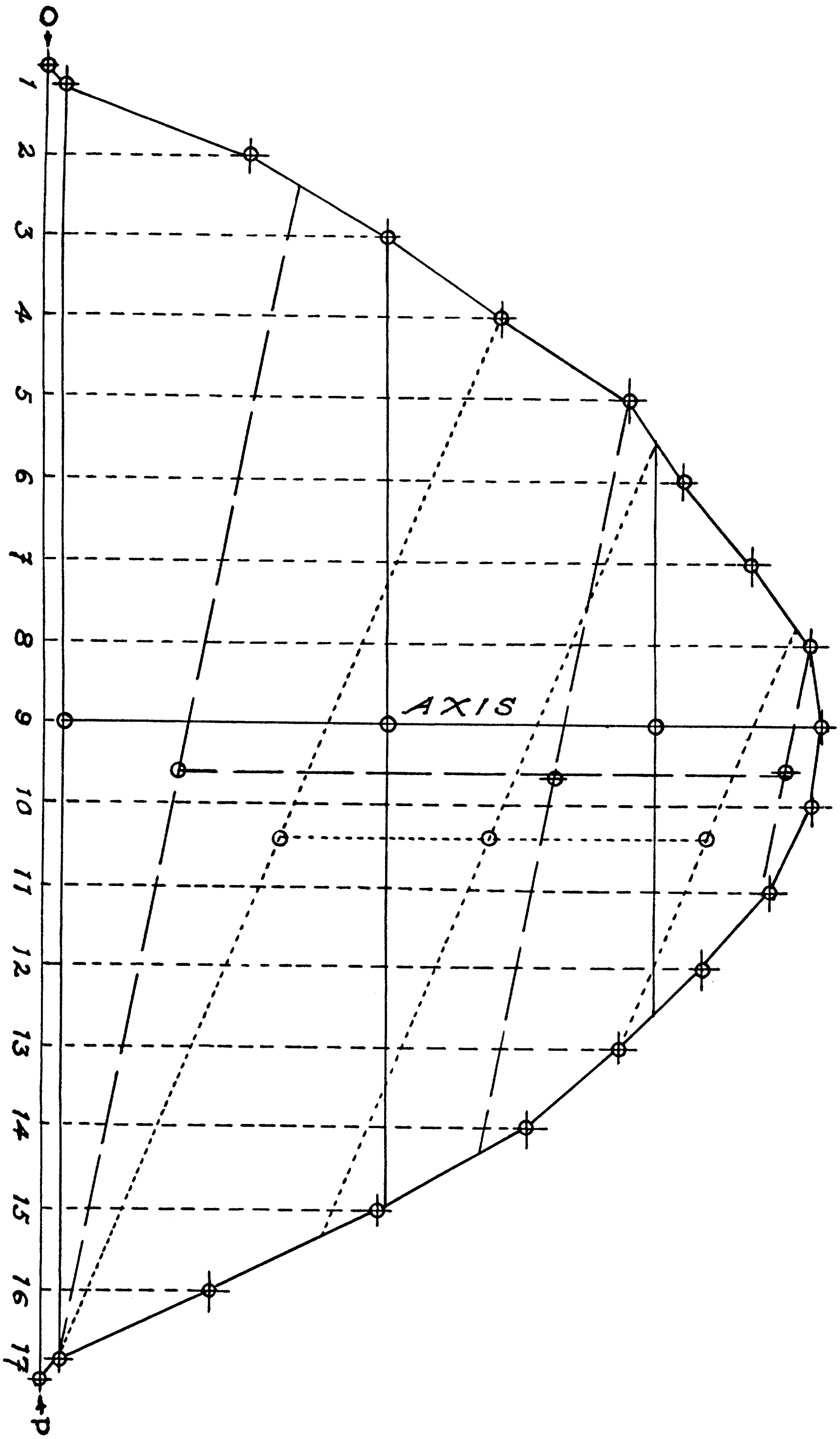

Figure 2

parabola of Figure 2 is then vertical and at the center of the stylobate (cf. Figs. 1 and 2), and further, the maximum rise is vertical and at the center of the stylobate.

In passing, a word may be said about the maximum rise (cf. Figs. 1 and 2). Its amount is 0.103 m. according to calculations from Balanos' drawings, and 0.1025 m. according to Balanos' table of measurements. As an Attic foot of the 5th century B.C. equals 0.328 m., 1 dactyl will equal 0.328/16, or 0.0205 m.; and 5 dactyls will equal 5×0.0205 m., or 0.1025 m. This last figure is within half a millimeter of the figure derived from Balanos' drawing, and corresponds exactly to the figure in his table. Surely the maximum rise of the curve was intended to be 5 dactyls of ancient measure and was to occur at the center of the stylobate.

A circle is an ellipse in which the major and minor axes are equal. Therefore, if the curve of the stylobate had been a portion of a big circle, as Professor Caratheodory's article leads one to suppose, and if the horizontal measurements of that big circle be drawn at 1 : 400, the resulting curve will be a portion of an ellipse—this is schematically represented in Figure 3, where, however, the horizontal distances have been drawn *not* at 1 : 400 but at 1 : 2 (1 to 9 equals twice 1′ to 9″, etc.). Note that the loci of the center points of the parallel chords are not parallel to each other—they pass through a common point (the center of the ellipse) within the curve.

Let us study Figure 3 a little more in detail. We see that the bigger the radius of the circle the farther the center recedes from the apex of the ellipse, Q Q′ Q″ and R R′ R″ becoming, at the same time, more nearly parallel to each other. The radius of the circle which passes through 1 and 17 is 5662 m. to give a rise of 0.1005 m. (cf. Fig. 4). In other words, the center is so far off, that Q Q′ Q″ and R R′ R″ (cf. Fig. 3) become practically parallel. In fact the curve is so flat that we cannot say definitely whether the loci of center points of parallel chords are parallel to each other or meet at a point situated within the curve and at a great distance from the apex. We may go even farther and claim that the loci may have met at a point outside the curve, provided the distance of that point from the apex of the curve be very great. The curve in this case is a hyperbola. About all that we can definitely assert is that the curve we are studying is one of the three conic sections. To determine which of the three, we must, we believe, return to Vitruvius and his *scamilli impares*. From him we learn that curves of stylobates were laid out by the method of the *scamilli impares*—a method which is much easier than the method of laying out either an ellipse (including the circle of big radius) or a hyperbola. Vitruvius does not say that he invented the *scamilli impares* method. But there are strong inferences in his treatise to indicate that the method was a tradition, perhaps of long standing. He mentions a host of writers on architecture, and among them Ictinus himself (Vitruvius, VII, intro. 12 and 13). Now, the method of laying out a curve by means of the *scamilli impares* produces a parabola. If, therefore, Ictinus used that method, which seems

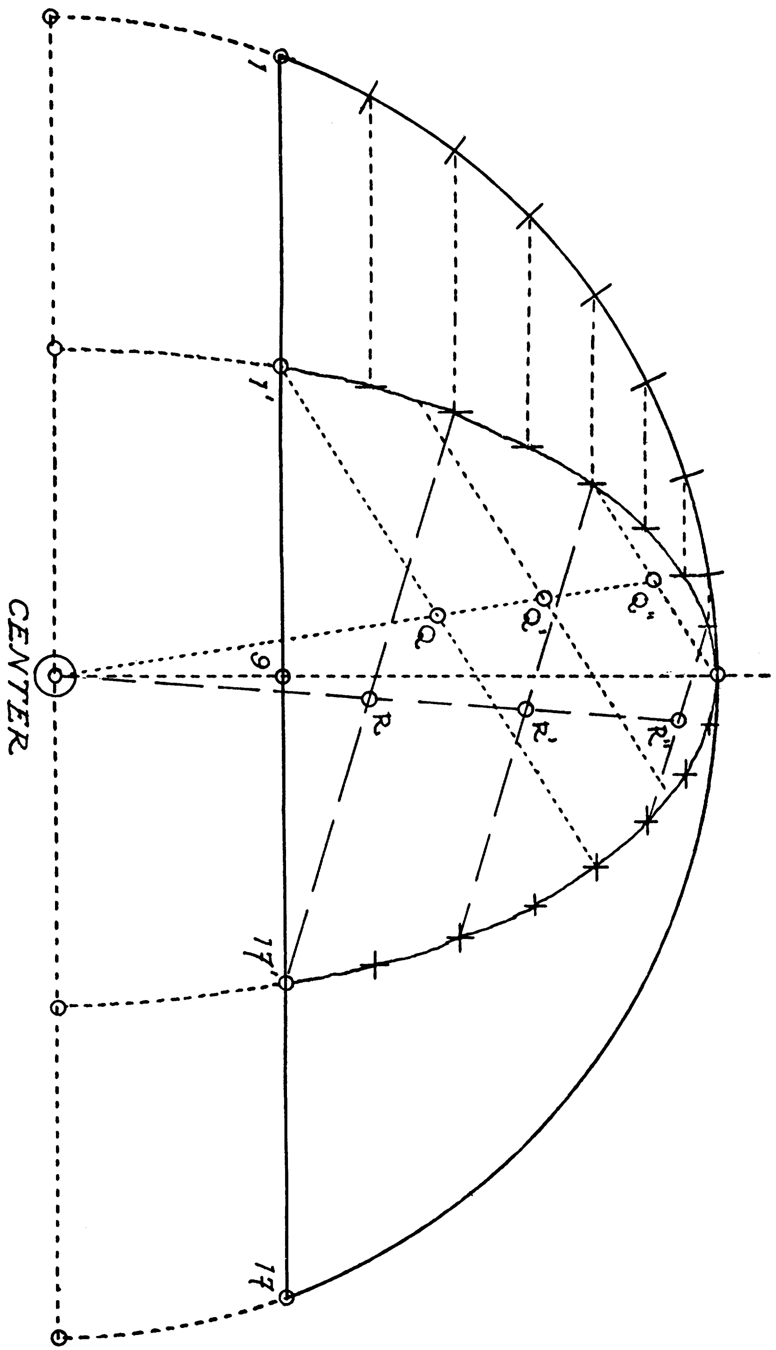

Figure 3

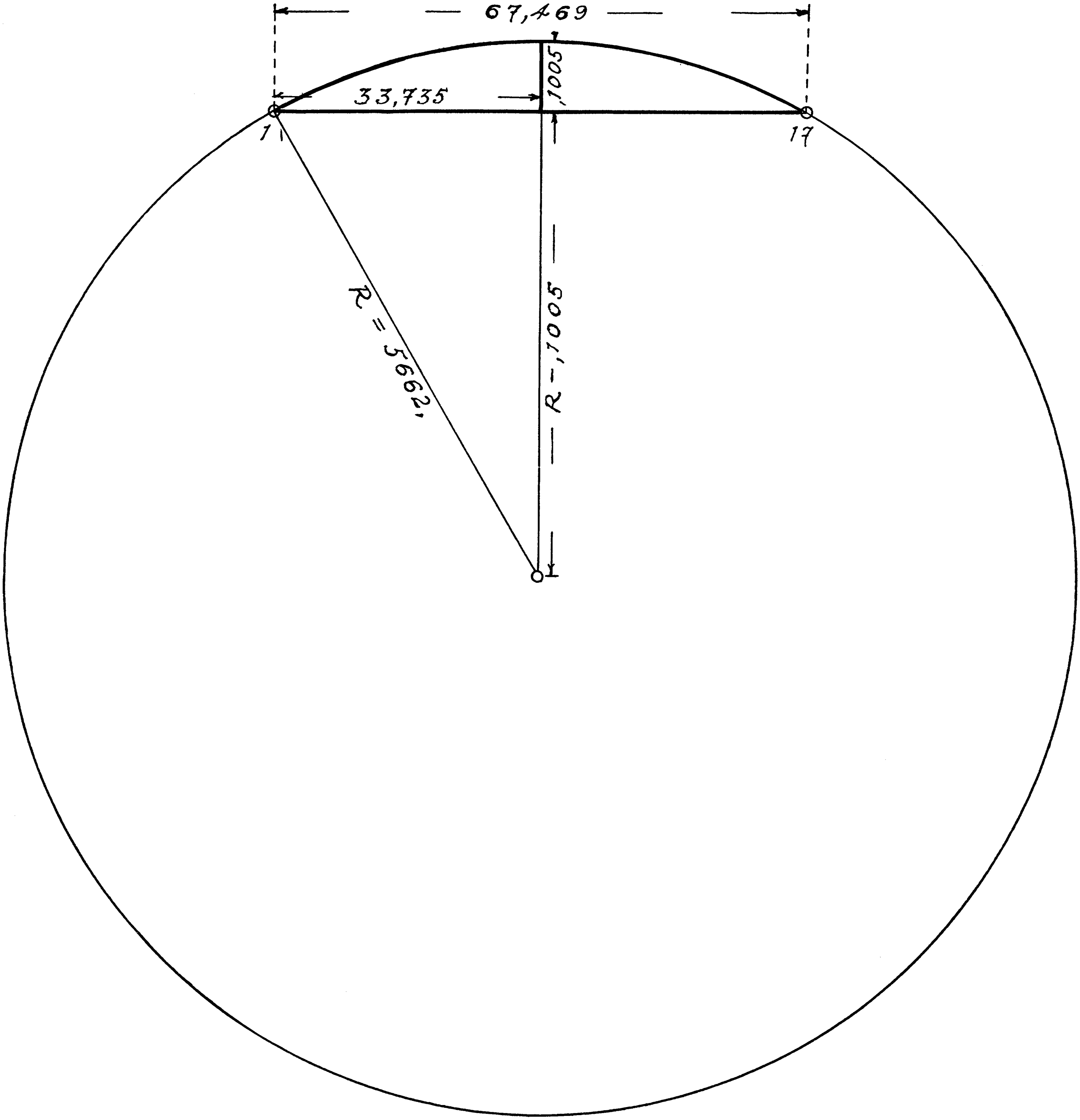

Figure 4

highly probably, then the curve of the north stylobate of the Parthenon was a parabola, as nearly as Ictinus could make it. Whether he *understood* that his curve was a parabola must, the writer believes, remain a matter of speculation. Perhaps all that he knew was that he could obtain his curve easily and quickly by the *scamilli impares* method.

Penrose, as has already been observed, advanced the theory, but without attempting to prove it, that the curve in question was probably a parabola. The writer's investigations make him believe that Penrose was close to the truth.

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