IDEA AND VISUALITY IN HELLENISTIC ARCHITECTURE

A GEOMETRIC ANALYSIS OF TEMPLE A OF THE ASKLEPIEION AT KOS

ABSTRACT

The author uses analytic geometry and AutoCAD software to analyze the plan of Temple A of the Asklepieion at Kos, revealing a circumscribed Pythagorean triangle as the basis for the plan's design. This methodology and its results counter earlier doubts about the application of geometry to Doric temple design and suggest the existence of an alternative to the grid-based approach characteristic of Hellenistic temples of the Ionic order. Appreciation of the geometric system underlying the plan of Temple A leads to a consideration of the role of visuality in Hellenistic architecture, characterized here as the manner in which abstract ideas shared by architects and scholars conditioned viewing and influenced the design process.

The Asklepieion on the island of Kos was a healing sanctuary and medical school of great importance throughout antiquity. It lies some 4 km southwest of the ancient polis of Kos, built on a terraced slope commanding impressive views of the sea. In its completed state, the complex consisted of three separate terraces connected by stairways, each supporting structures from various periods (Figs. 1, 2).

By the middle of the 3rd century B.C., the sanctuary's three terraces were constructed. On the lower terrace, a Π-shaped Doric stoa with adjoining rooms was built to enclose an approximately 47 x 93 m space. Major architectural features on the middle terrace included an altar, replaced by a more monumental version in the following century, and temples dedicated

1. I wish to thank Andrew Stewart for his constructive criticisms and for taking an interest in my arguments, which are all the stronger for our conversations. I am indebted to Fikret Yegül and Diane Favro for their devoted attention to this study from inception to completion. I am also grateful to Erich Gruen and Crawford H. Greenewalt Jr. for their generous encouragement of this project following an initial presentation of my arguments at an Art History and Mediterranean Archaeology Colloquium at the University of California, Berkeley, in April 2005. All drawings and photographs are my own.


4. A centrally placed propylon on its north wing served as the monumental entrance to the sanctuary; Schazmann and Herzog 1932, pp. 47–48.

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to Asklepios and Apollo (Temples B and C, respectively; see Fig. 1). On the upper terrace, a Π-shaped stoa of timber construction balanced the stoa of the lower terrace.

The first half of the 2nd century B.C. witnessed changes and additions to the upper terrace that resulted in a new character for the sanctuary as a whole. To connect the upper terrace with the rest of the sanctuary below, a new grand staircase created a dominant central axis (Fig. 3). In addition, a new marble stoa replaced the earlier timber structure. In the center of the approximately 50.4 × 81.5 m space enclosed by this stoa, a marble Doric temple of Asklepios was begun as early as 170 B.C., today referred to as Temple A (Figs. 2–5) to distinguish it from the earlier temple of Asklepios (Temple B) on the terrace below. Axially placed before the staircase overlooking the middle and lower terraces, Temple A became the dramatic visual focus of the entire Asklepieion.

The choice of the Doric order for Temple A is an archaism. While Doric stoas continued to be common in all areas of the Greek world down through the Hellenistic period, Asia Minor and the nearby islands reflect

5. Important utilitarian features, such as a springhouse and wells located along the retaining wall for the upper terrace, were also found on this level. For these features, as well as the 2nd-century monumental altar, see Schazmann and Herzog 1932, pp. 25–31, 34–39, 49–51, 60, 73, pls. 12–14.

6. For the timber portico and its later marble replacement, see Schazmann and Herzog 1932, pp. 14–21, figs. 15–17, pl. 9; Coulton 1976, pp. 9, 62, 75, 98, 109, 112, 149, 159, 171, 246, fig. 74.


8. Schazmann and Herzog 1932, pp. 3–13, figs. 3–14, pls. 1–6. The temple is oriented 25 degrees west of north. Built on a foundation of limestone, the superstructure of the temple is constructed of marble throughout with the exception of courses of poros limestone blocks in the interior walls of the naos.

Figure 1. Asklepieion at Kos, view of the middle and lower terraces from the upper terrace, with the remains of the 3rd-century B.C. Temple of Asklepios (left), the 2nd-century B.C. restoration of the altar (center), and the 2nd–3rd-century A.D. restoration of the Temple of Apollo (right).
Figure 2. Asklepieion at Kos, view of the remains (in situ) of the upper terrace complex from the southeast, looking toward Temple A, with the stoa in the foreground.

Figure 3. Asklepieion at Kos, restored plan of the upper terrace complex with Temple A.

9. In addition to Pytheos and Hermogenes, Vitruvius mentions the architect Arkesios, who perhaps dates to the 3rd century. For Arkesios, as well as the convincing and still much overlooked arguments against the common Vitruvian conception of a "decline" of the Doric order in the 4th century B.C., see Tomlinson 1963.


A predilection for the Ionic order for temple architecture. As Vitruvius indicates, architects such as Pytheos and Hermogenes bolstered this preference with a theoretical justification (Vitr. 4.3.1–2). Furthermore, as its measurements demonstrate, Temple A was traditional in its omission of...
the kind of novel modifications and “optical refinements” characteristic of the Parthenon. Foregoing also the interesting and easily detectable schemes of Ionic temples associated with Pytheos and Hermogenes (Fig. 6), the temple would seem to have been a strictly conventional reapplication of the Doric order in the 2nd century B.C.

Yet the straightforward character of Temple A may represent only a part of its story. As I argue below, a geometric analysis of its measurements reveals the use of a compass in constructing the interrelationships of architectural elements in plan according to circumferences.11 The diameters of these circumferences share a simple arithmetical relationship based on the whole-number proportions of a 3:4:5 Pythagorean triangle, rather than a more strictly geometric relationship pertaining to irrational numbers like √2 or √3, or their fractional approximations. The geometry of the temple’s plan is therefore very simple, and is not to be confused by the analytic geometry required to substantiate it.

The presence of theoretical circumferences concealed within the building’s features raises interesting questions about the nature of the Doric design process on a Hellenistic architect’s drawing board. That such an underpinning is found in only a single (albeit prominent) example of Greek temple architecture, as opposed to the more widespread approach of grid patterns, does not detract from its significance. As I will discuss, the uniqueness of circumferential relationships in a temple plan—as opposed to the kind of orthogonal relationships that temples of the Ionic order permit—relates to a dearth of specifically Doric temples during the Late Hellenistic period. The interesting geometry in Temple A demonstrated here exemplifies an important architectural tenet that we might term “cryptomethodic,” referring to the systematic features of the design process that cannot be appreciated through casual observation, but may be recovered only through detailed study.

11. For an excellent discussion of plans in ancient architecture, see Haselberger 1997.
Figure 5. Measured state plan of Temple A according to material and trace remains, shown without the exposed limestone masonry of the foundations
IDEALISM AND HELLENISTIC VISUALITY

Before turning to a technical discussion, I will first address the very premise that an ancient architect should design a building based on geometry that does not correlate experientially with the final product. It is important to state from the outset that architects of the Hellenistic world thought about their buildings in terms different from those used by architects today. We know from Vitruvius that Greek architects called their plan, elevation, and perspective drawings ἰδέα (Vitr. 1.2.1–2), corresponding to the notion that Platonic idealism uses in reference to the transcendent ideas (or forms) that are thought to be the ultimate reality underlying the perceptible objects of the everyday world. As Lothar Haselberger has admirably observed, the correspondence between the philosophical and architectural meanings is not casual, and the full implications of this correlation have yet to be appreciated in studies of ancient architecture.

The rhetorical manner in which Plato sometimes discusses this idealist vision can seem quite foreign to our own way of thinking, as when he presents Socrates’ argument that couches manufactured by artisans can only imperfectly imitate an archetypical couch existing in a realm beyond our senses (Resp. 10.596e–597e). Yet it is unfair to reduce Plato’s conception to these isolated metaphors and parables, and the lack of any clearly stated unifying theory of ideas in Plato’s work should draw our attention instead to the more general importance of mathematics as a model for systematic and hierarchical methods of penetrating to the ultimate realities of the universe in Plato’s idealism. Perhaps the most articulate expression of this way of understanding is the well-known passage in which Socrates, after guiding an uneducated slave through a geometric proof, concludes that eternal truths lie beyond our embodied experiences in the world (Meno 82b–86c). According to the Platonic model, it is the theoretical rather than the sensory that is privileged.

What brings this discussion to bear on the question of underlying geometric systems in architectural plans is what J. J. Pollitt terms the “scholarly mentality” of the Hellenistic age. Perhaps originating in the

12. The following considerations pertain not only to the ancient world, but more generally to how culturally based understandings of the world anticipate the way in which objects are viewed and visually constructed. For a discussion of this idea in the contexts of Cartesian perspectivalism, early modern painting, and 19th-century photography, see Jay 1988, esp. pp. 16–17. A definition of visuality offered by Norman Bryson (1988, pp. 91–92) has recently been evoked by Jas Elsner in his new study of visuality in a classical context: “the pattern of cultural constructs and social discourses that stand between the retina and the world, a screen through which . . . Greek and Roman people had no choice but to look and through which they acquired (at least in part) their sense of subjectivity” (Elsner 2007, p. xvii). I focus here not on subjectivity and what texts and images can tell us about visibility in the classical world, but rather on how the geometric underpinning of Temple A relates to ways of seeing that were culturally and socially conditioned.

13. Haselberger 1997, esp. pp. 77, 92–94, and primary and secondary sources cited. Hannah Arendt (1958, p. 90) offers an excellent philosophical articulation of how Platonic ideas relate to notions of models and measures, which may be useful for framing the conceptual connection between ideas

and geometrically based drawn models in architecture: “For the transformation of the ideas into measures, Plato is helped by analogy from practical life, where it appears that all arts and crafts are also guided by ‘ideas,’ that is, by the ‘shapes’ of objects, visualized by the inner eye of the craftsman who then reproduces them in reality through imitation. This analogy enables him to understand the transcendent character of the ideas in the same manner as he does the transcendent existence of the model, which lives beyond the fabrication process it guides and therefore can eventually become the standard for its success or failure.”

intellectual ambience of the Library at Alexandria, a taste for didactic displays of abstruse knowledge came to strongly characterize Hellenistic art and literature. A notable feature of works appears to have been the deliberate potential for simultaneous appreciation from both common and erudite perspectives. In architecture, in particular, this tendency is found in examples such as Pytheos’s Temple of Athena Polias at Priene (Fig. 6, left), in which the masses might marvel at its surface qualities, while those who knew the building’s proportions could understand its plan as an expression of mathematical precision.15

Vitruvius, whose text depends in part upon the writings of earlier Hellenistic architects, exemplifies this scholarly emphasis. He insists that an architect’s background in disciplines like geometry, music, and astronomy is requisite (Vitr. 1.1.4, 8–10), a claim that he backs up at times with pretentious displays of erudition. Sometimes his eagerness to show his knowledge exceeds his command of the material that he discusses, as when he credits Plato with the demonstration of the doubling of the square, which he follows immediately with an introduction to the Pythagorean theorem, without realizing that both of these theorems illustrate an identical principle of proportion (Vitr. 9.Praef.4–7).16 Despite such limitations, he

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Figure 7. Plan of a Latin theater according to Vitruvius's description

The plan of the (Latin) theater itself is to be constructed as follows. Having fixed upon the principal center, draw a line of circumference equivalent to what is to be the perimeter at the bottom. In it inscribe four equilateral triangles at equal distances apart and touching the boundary line of the circle just as the astrologers do in a figure of the twelve celestial signs when they are making computations from the musical harmony of the stars. From these triangles, select the one whose side is closest to the scaena and in the spot where it cuts the curvature of the circle let the front of the stage be located. Then draw through the center a parallel line set off from that position to separate the platform of the stage from the space of the orchestra. . . . The wedges for spectators in the theater should be divided so that the angles of the triangles that run around the circumference of the circle may provide the direction for each flight of steps between the sections up to the first curved cross-aisle. Above this, the upper wedges are to be laid out with aisles that alternate with those below. The angles at the bottom that produce the directions of the flights of steps will be seven in number, and the remaining five angles will determine the arrangement of the scaena. In this way the angle in the center ought to have the “palace doors” facing it and the angles to the right and left will designate the position of the doors for “guest chambers.” The two outermost angles will point to the passages in the wings.17

And for the Greek theater (Vitr. 5.7.1–2):

In Greek theaters some things are done differently. First, in the bottom circle, while the Latin theater has four triangles, the Greek has three squares with their angles touching the line of circumference. The limit of the proscenium is determined by the line of the side of the square that is nearest the scena and cuts off a segment of the circle. Parallel to this line and tangent to the outer circumference of the segment, a line is drawn that delineates the front of the scena. Draw a line through the center of the orchestra and parallel to the direction of the proscenium. Centers are marked where it cuts the circumference to the right and the left at the ends of the half-circle. Then, with the compass fixed at the right, an arc is described from the horizontal distance at the left to the left-hand side of the proscenium. Again, with the center at the left end, an arc is described from the horizontal distance at the right-hand side of the proscenium. . . . Let the ascending flights of steps between the wedges of seats, as far up as the first curved cross-aisle, be laid out on lines directly opposite the angles of the squares. Above the cross-aisle, the other flights are laid out between the first. At the top, as often as there is a new cross-aisle, the number of flights of steps is always increased by the same amount.¹⁸

These prescriptions are not easy to follow, and it would be tempting to dismiss them as indicating a fussy outlook on the part of Vitruvius if not for the fact that these geometric constructions were applied in surviving Greek and Roman theaters.¹⁹ The prescriptions pertain to a basic geometry of forms such as equilateral triangles or squares, rather than considerations

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¹⁹ For the Greek material, see Isler 1989. For a discussion of questionable scholarly attempts to match Vitruvius’s description of Roman theater design to later Roman theaters, see Sear 1990; 2006, pp. 27–29.
based on irrational numerical relationships. In the case of both theater types, the cryptomethodic patterns described arguably would not contribute to any visible harmonic relationships, nor would most ancient visitors have been likely to perceive them. Certainly, there are less theoretically grounded ways of determining the locations of radial stairways, boundaries, and doorways, and it would seem more sensible to design forms based on simple intuition and functional criteria. On the other hand, our very concern with these issues may be a consequence of an inherently modern prerequisite that the design process directly correlate with sensory experience. For the privileged few in the Hellenistic world who could read and understand such passages, there was value of a different kind: the value of discourse. Furthermore, for material pertaining to a world that validates the independence of underlying ideas, our own privileging of the tangible properties in the final built form is arguably misplaced.

Vitruvius’s reference to the drawings of astrologers reveals a significant interdisciplinary issue at work in such architectural ideas. Given the scholarly interests of Hellenistic architects, there is no reason to believe that the practice of architectural drawing developed in reference solely to the designs of buildings. Another drawn construction that Vitruvius describes in detail is the Greek ἄναλημμα, which was the graphic reference for solar declinations that served as the basis for sundials (Vitr. 9.1.1, 9.7.2–7). He provides an algorithm for the drawing, which has allowed for its reconstruction as a markedly circumferential design (Fig. 9).20 In this curvilinear quality, the analemma provides intriguing general comparisons with the circumferential geometry underlying the design of Temple A at Kos proposed below. Interestingly, Berossos the Chaldean, whom Vitruvius credits with the invention of the semicircular sundial, moved to Kos and established there a school of astronomy following Alexander the Great’s conquest of Mesopotamia (Vitr. 9.2.1, 9.8.1). In the course of the 3rd century, Berossos’s school amalgamated with elements of the Koan medical school to establish the discipline of medical astrology, concerned particularly with the moment of conception as the basis for casting nativities (Vitr. 9.6.2).

I do not argue that there was any symbolic connection between Temple A and the analemma, let alone some sort of mystical value. The study of architectural iconography is an inexact science, and we cannot lose sight of the fact that here we deal with a design that pertains solely to the architect’s drawing board; it is only the circumferential approach that is similar, underscoring the possibility of shared ways of envisioning (and thereby drawing) forms among architects and those concerned with astral phenomena as well as geometry. As I demonstrate below, the curvilinear element in the underpinning of Temple A pertains not to solar declinations, but rather to a Pythagorean triangle. In this aspect, it is similar to the ways in which Greeks and Romans began their theaters with squares or triangles, and consistent with an outlook characteristic of the ways in which educated men of the Hellenistic period thought. While the drawn plan expresses an eternal and abstract form of the idea, the final built form brings that idea into presence in ways that need not readily unveil its underlying mathematical truth to the senses.

In the Hellenistic period, then, visuality in architecture was constituted not solely by the perceptions of the casual viewer, but also by the

epistemological certainty of geometry. Framed by a monarchically sponsored scholarly agenda and the resulting practices of visualizing form according to geometry, modes of visual representation established the starting point for form in disembodied abstractions that were subject to mathematical rules or norms. In this way, the squares underlying the placement of empirical features within the curvilinear Greek theater were patently real. Similarly, the circles (and the Pythagorean triangle that gives measure to their proportions) underlying the experiential rectilinear forms of Temple A at Kos discussed below relate to a primary consideration: the geometry that defines the visuality of the building by constructing its eternal idea. The square, circle, triangle, and other shapes are the ideas of nature that engender the visible things in the world, be they a theater, a temple, or even a human body (Vitr. 3.1.3). As in Vitruvius’s discussion of theaters, the drawing of a building may begin with geometry alone, and only through the process of design arrive at the final form. In further support of these observations, I argue that the design for the plan of Temple A at Kos similarly began with the drawing of a Pythagorean triangle, from which the design and construction evolved into a completed expression that continues to reflect its origin, however imperfectly.

QUESTIONABLE METHODOLOGIES

The very suggestion of a hidden system within an architectural plan tends to touch a raw nerve among archaeologists and architectural historians alike. Far from striking an innovative note, such an approach falls squarely within a tradition that has so tried the patience of readers that it may one day risk outright exclusion from mainstream scholarly research. Before proceeding, it is necessary to briefly address this circumstance.
In a penetrating essay on the Parthenon, Manolis Korres offers a markedly negative assessment of efforts to present that celebrated building as an expression of ideal numerical relationships and harmonious proportions. Characterizing such studies as "pseudo-science," Korres notes the disturbing tendency to argue theories that contradict the reality of the building. According to him, approaches include impudent suggestions of error in the temple's construction and published measurements; the reliance of proposed geometric theories on inaccurate, small-scale drawings rather than the degrees of magnitude found in the actual building; the inability to credibly correlate the proposed geometric shapes with analytic geometry; and the obsessive or even mystical motivations that may underlie such studies in the first place. Observations like these have articulated and, justifiably, perhaps even reinforced general reservations about the rigor and value of metrological and geometric studies of architecture in various periods and locations in the ancient Mediterranean world.

Although Korres's remarks may provide salutary caution to future studies, we may not entirely benefit from severe marginalization of geometric analysis in Greek temple design. For one thing, the Parthenon antedates the use of scale drawings in architectural planning. In buildings of the Hellenistic period, when such drawings were used (Fig. 6), questions concerning the geometric basis of plans become considerably more applicable. Writing at the close of the Hellenistic period, Vitruvius (1.2.1–2) clearly describes Greek temple design process in terms of τάξις, or the creation of a quantitative geometric system, and διόθεσις, the placement of architectural elements according to that established geometry. While Vitruvius's comments cannot comprehensively represent Hellenistic practice,

21. Korres's statements (1994, pp. 79–80) reflect similar misgivings going back at least as far as William Bell Dinsmoor (1923a, 1923b). Readers less familiar with scholarship on Greek architecture might not be aware of the degree of esteem attached to Korres and his views, something that comes across particularly in less formal settings. In an aside during a recent public lecture at the Art Institute in Chicago, for example, Jeffrey Hurwit referred to Korres as a "genius" (Hurwit 2005). In my own view, the remarks of Korres referenced in this study are characteristically incisive, and it is these remarks and their implications that I invoke as a background for the methodology used in my analysis of Temple A at Kos. To be clear, I in no way draw any meaningful comparison between Temple A and the Parthenon. With its markedly greater sophistication in execution and details, the Parthenon is an expression of a completely different mentality from what we find in Temple A, and is the product of a different era. The following discussion pertains to methodological issues that carry implications for any geometric analysis of ancient temple architecture, and not to the architecture of the Parthenon per se.

22. Korres 1994, pp. 79–80. Similar criticisms may be directed toward the study of Greek architectural environments by C. A. Doxiadis (1972), including his analysis of the Asklepieion at Kos. Lacking both a proper trigonometric analysis and convincing identification of salient architectural features pertaining to his proposed geometry, Doxiadis suggests that the sanctuary's structures from various eras were intended to relate to one another through sight lines established at angles related to the "golden section." He does not attempt an analysis of Temple A. See Doxiadis 1972, esp. pp. 125–126, fig. 77.

23. In support of his theory for "facade-driven" Doric design in the 5th century B.C., Mark Wilson Jones (2001, p. 678) advocates a "general rule [that] ancient architects exploited geometry for resolving details ... but not for the composition of whole buildings unless they were concentric or partially concentric in plan." Wilson Jones's own acceptance of such complex geometry and numerical systems in circular buildings and other structures (2001b) has, in turn, elicited doubts extending even to material of the Roman period; see, e.g., Yegül 2001.

24. A particularly forceful argument in favor of detailed architectural drawings for at least one Classical-period Athenian building, the Propylaia, is Dinsmoor Jr. 1985. For opposing views on the introduction and role of drawn plans in Greek architecture, see Haselberger 1997, p. 83.

25. For the development of scale plans during the Hellenistic period and alternative modes of architectural design in earlier periods, see Coulton 1988, pp. 51–67.

26. For complexities arising from Vitruvius's use of Greek terminology in this passage, see Frézouls 1985, esp. p. 217.
his stated reliance upon Greek architectural writers arguably merits continued investigations into the geometric underpinnings of Hellenistic buildings. In addition, Vitruvius’s apparent adherence to grid-based approaches in Ionic temple design might elicit inquiry into procedures that he does not elaborate upon: in the Doric order, where intercolumnar spatial contractions do not lend themselves to an orthogonal grid, how might the geometric constructions of *taxis* differ?

As a privileged monument, furthermore, the Parthenon will continue to be a favored object of attention for numerous lines of inquiry despite condemnations of particular approaches. Dating from the 19th century onward, however, are too many volumes of published archaeological reports with scientific measurements pertaining to buildings about which we still know relatively little. To allow these to gather dust or occupy unused electronic storage space, instead of reaping what the laudable efforts of their excavators can tell us about ancient architectural design, will benefit neither archaeologists nor architectural historians. Furthermore, accusations of “intellectual totalitarianism” directed at proponents of geometric analysis could serve only to curtail productive discussion.

Rather than framing various outlooks as scientific or mystical, as rational or obsessive, we might instead see observations such as those of Korres as an opportunity to reevaluate the methodologies employed in proportional and geometric analyses. In addition, an inclusive view may open us to methods that allow for scientifically sound analyses that, in turn, solidify our understanding of Hellenistic temples. Finally, a responsible, mathematically rigorous, and computer-based approach to geometric analysis will help us build upon and refine the criticisms, rules, and expectations of similar studies.

The present study uses analytic geometry and vector-based AutoCAD (or CAD) software to analyze the geometric underpinning of the design of Temple A at Kos. In the course of this analysis, I also consider questions surrounding the perceived limitations of studies that attempt to unveil hidden numerical and geometric systems. In order to avoid the inevitable distortions of proportional and geometric relationships that look correct only when overlaid on a plan drawn to reduced scale, my study instead directly relies on the building’s published measurements. In other words, the proposed geometric system is now mathematically verifiable rather than intuitive, and is grounded in computation. So that we may furthermore ensure both mathematical accuracy and the relationship between the numerical systems and the concrete, graphic form of the revealed geometry, the calculations have been verified through the use of AutoCAD. CAD is not a requisite for this study, but merely a convenient tool that may allow researchers and readers a simpler recourse to the measurements of proposed relationships in an architectural form; it is ultimately the calculations themselves that demonstrate the geometry. This combined Cartesian and computer-based method carries the potential of standardization for future studies, allowing for a truly scientific approach in which results may be replicated to confirm their veracity. Provided that an analysis such as this one relies upon previously published numbers rather than one’s own measurements, we may now set aside suspicions of personal agenda and have confidence in the objectivity of the process.

27. See the comments of Thomas Howe in Howe and Rowland 1999, pp. 5, 14, 149.

28. See Vitruv. 4.3.1–8, where he expresses his indebtedness to the Ionic tradition of Hermogenes by characterizing the Doric order as deficient, leaves the issue of columnar interaxes unexplained, and focuses on elevations at the expense of any discussion of plans. Wilson Jones’s related notion of “façade-driven” Doric design (2000b, pp. 64–65; 2001) is discussed below; see also n. 23, above.

ANALYSIS OF THE BUILDING

The Metrology

Before discussing the nonorthogonal dimensions revealed through analysis, we should consider the temple's general measurements. From the 5th century onward, it was a common rule of thumb that the width:length ratio of a Doric temple's plan (including the euthynteria) should match the number of columns on the short and long sides of its peristyle. With six columns along its front and rear and eleven along its flanks, Temple A appears to be no exception (Fig. 10). In plan, the temple's overall dimensions are 18.075 x 33.280 m, a differential of only 0.4% from a proper 6:11 ratio. A simple adjustment, such as a 0.143 m reduction of the overall length or a 0.078 m increase in the width, would result in a perfect whole-number ratio.

Scholars usually account for such "errors" by citing constructional inexactitude and adjustments, as well as centuries of exposure to the elements. Other slight irregularities found throughout the temple might support this notion of a difference between the theoretical design and the actual built form (Fig. 5). For example, there are slight variances in the thickness of the eastern and western naos walls (1.028 and 1.016 m, respectively) and in the distances from the exterior of these walls to the edges of the stylobate (3.313 and 3.380 m, respectively). As a result, the naos is not centered on the stylobate.

Although factors such as imperfect masonry and deterioration over time are plausible explanations for such disparities, additional considerations deserve emphasis. If it is the architect's design to begin with a proper 6:11 plan, other features might complicate the maintenance of perfect proportions in the final built form as the construction progresses. In the end, there will be a set of measurements that are necessarily interrelated, such as the widths of the krepidoma and the overall dimensions of

31. Schazmann and Herzog 1932, pl. 2.
32. For the problem of the difference between the abstract vision of the architect and the final product, see Wilson Jones 2000b, pp. 11–14; Dwyer 2001, p. 340.
33. Schazmann and Herzog 1932, pl. 2.
34. The excavators of Temple A reason that this lack of symmetry is a result of earthquakes that have shifted the entire naos and pronaos eastward, an explanation that I find unconvincing; see Schazmann and Herzog 1932, p. 6. Based on my on-site analysis, there are differences in the limestone foundations on the eastern and western sides of the naos. While the masonry on the eastern side runs in courses that are parallel with the long walls of the naos, that on the western side runs in courses that are roughly perpendicular to these walls. In addition, the joints on the western side are tighter than those toward the east. These divergent tendencies continue into the raised foundations of the naos itself, where the two separate approaches meet at a line west of the central axis of the naos.

The difference should indicate that separate crews were responsible for laying the limestone foundations on either side of the temple. More plausible than the eastward shift of the entire cella is that one crew committed a minor error in establishing the eastern limit of the stylobate, or possibly the euthynteria, resulting in a distance from the cella that is 0.067 m less than that found on the western side. This interpretation is supported by the nearly equal distances of 4.43 and 4.435 m from the naos walls to the outer edge of the euthynteria on the western and southern sides, as opposed to 4.368 m on the eastern side. If we maintain that approximately 4.435 m was originally intended for the eastern side as well, the result would be a more balanced design than if the entire cella originally lay on the central axis of the temple in its present dimensions. I therefore favor human error as opposed to natural causes for the lack of symmetry in the temple's measurements. Such errors can and do occur in the laying of foundations, affecting the placement of elements in the superstructure.
Figure 10. Restored plan of Temple A, with measurements of the colonnade axes (M = meters, F = Doric feet, T = triglyph width modules)
the stylobate, which in turn relate to the sizes of the paving slabs and the spacing of the columns they support. The area where the relative proportions of the various parts are subject to modification is the conversion from the abstract units of the drawing board (such as 6 x 11) to actual metric values. In determining specifications, certain distances must be privileged while others must be adjusted to the space allotted them. Considerations such as the specific measurements of the paving slabs, for example, may ultimately result in a slight departure from the integral proportions of the architect’s original drawn plan.

One method of accounting for the overall and individual dimensions of a building is a metrological analysis. A recent study proposes that the architect of Temple A first worked out the overall dimensions according to a specific metrological system.35 Only thereafter were the usual corner contractions of the Doric order worked out, resulting in adjustments to the dimensions of the theoretical plan. This theory, however, relies upon the identification of a 0.305 m “foot” as the common unit underlying the temple’s metrological system. Simply put, there exists no such unit of measurement in the ancient Greek world, a fact that the theory’s authors contend with by advocating greater flexibility in our understanding of Greek metrology.36

Instead of suggesting new units of measurement, we may consider the issue of commensuration. For Doric temples, specifically, Wilson Jones makes a detailed case for a modular system, at least for 5th-century examples.37 According to this theory, the width of a standard triglyph expresses the module that establishes commensurability throughout various elements of the building.38 The triglyph module itself commonly corresponds to a 5-dactyl multiple of a standard foot (e.g., 25 or 30), with a dactyl equal to 1/16 of a foot in accordance with Greek metrological standards.39

In Temple A, measurements for the remains in situ are available for the central columnar interaxis and the western half of the columns on the rear of the stylobate, as well as four columns along the western lateral colonnade (Fig. 5). At the rear, the addition of 5.793 m for the missing eastern half (5.793 + 3.080 + 5.793 m) results in a length of 14.666 m for the entire axis (Fig. 10). For the long sides, the temple’s excavators posit columnar interaxes of 3.05 m based on the remains in situ and a consideration of the triglyphs and metopes, which measure 0.61 and 0.915 m, respectively (Fig. 11).40 Thus the one preserved interaxis of 3.034 m (see Fig. 5) would represent an unintended departure from the theoretical constant of 3.05 m, and we may thereby restore the theoretical lateral axes, excluding the contracted corners, to a length of 24.4 m (8 x 3.05 m), as in Figure 10. Therefore, the 24.4 m axes of the lateral colonnades and the 14.666 m axes of the front/rear colonnades would equal 40 and 24 integral units, respectively, of a value equal to 0.61 m (Fig. 10).41

36. Pettit and De Waele 1998, esp. p. 62. In an earlier essay, J. J. de Jong claims to have analyzed the measurements of Temple A, but offers no discussion or results pertaining to his analysis; see de Jong 1989, esp. p. 104, fig. 3.
37. Wilson Jones 2001. Regarding the possibility that such a system could have endured into later periods, see the author’s comments on p. 697, n. 107 (in response to Coulton 1983).
40. These measurements are based on three surviving fragments of the frieze; see Schatzmann and Herzog 1932, pp. 10–11.
41. 24.4/40 = 0.610 m; 14.666/24 = 0.611 m.
Significantly, Temple A's triglyph widths also measure 0.61 m. A simple calculation shows that this value equals 30 dactyls of a 0.325 m "Doric" foot. This triglyph width shares a 2:3 relationship with the standard metope, a 1:5 ratio with the average interaxial spacing of the lateral colonnades, and a 1:24 ratio with the axis of the facade colonnade—all typical proportional relationships according to Wilson Jones's study (Figs. 10, 11). It is also interesting that these distances of 40 and 24

42. Schazmann and Herzog 1932, pp. 10–11.

43. 0.610 m/30 = 0.02033 m. Since a foot divides into 16 dactyls, 0.02033 m × 16 = 0.325 m. Varying between 0.325 and 0.329 m, the Doric foot has been known since Wilhelm Dörpfeld's study (1890) of the late-5th-century inscription relating the expenses involved in the construction of the Erechtheion, combined with measurements taken from various buildings on the Acropolis and throughout Attica. The investigations of William Bell Dinsmoor (1961), who coined the term Doric foot, confirmed a value of 0.326 m. See also Wilson Jones 2000a, p. 75; 2001, p. 689. The metrological relief from Salamis, previously thought to represent a system based on a 0.322 m foot according to the measurements of Ifigenia Dekoulakou-Sideris (1990), has now been convincingly shown by Wilson Jones (2000a) to represent a system based on a 0.3275 to 0.3280 m Doric foot. For the divisibility of the triglyph module into 20, 25, 30, etc., dactyls, see Wilson Jones 2001, p. 690.

triglyph modules correspond to precisely 75 and 45 Doric feet.\textsuperscript{45} In translating the drawn plan to the actual dimensions of the building and its features, then, it is reasonable to theorize that the architect may have privileged the colonnades of the facade and rear, establishing their axes of 45 Doric feet. Through this magnitude, a 3:5 ratio finds the 75-foot measurement for the axes of lateral colonnades (Fig. 10). This latter dimension divides into eight intercolumniations, each of which subdivides into two half triglyphs, one whole triglyph, and two metopes (Fig. 11). Furthermore, the distance separating the end columns of these axes establishes the measurements for the contracted corners, and the remaining three interaxial distances of the facade and rear colonnades could be set according to the criterion of incremental widening toward the center. In varying the dimensions of the individual paving slabs in accordance with this irregular column spacing, the total dimensions of the stylobate are established, and the widths of the stereobate and euthynteria are set according to the remaining distance necessary to maintain the 6:11 ratio of the overall plan.

To insist upon this explanation, however, is to treat Temple A as we have the Canon of Polykleitos, resulting in yet one more plausible theory that can never be proven. There are too many types of metrical units, too many ways of measuring, and too many rationales for us to induce conclusively a guiding metrological system. What is lacking in such approaches is not so much a reasonable correspondence to a pattern of numbers, such as whole-number ratios, but rather something outside of the buildings themselves that might verify the significance of those numbers, such as a primary source or a basis in Euclidian geometry. While Vitruvius validates the relevance of the triglyph module, the case for how this system relates to large-scale distances must remain provisional; in this regard, we may wonder, for example, why the 40 integral units of the lateral colonnades exclude the corner interaxials. In addition, the modular theory as applied to Temple A cannot address a central aspect of design that is unrelated to the trabeation: the placements of the walls of the naos and pronaos in relation to the overall plan. We must therefore explore other methods of analysis in seeking to substantiate a theory for the underlying logic of the building’s design.

The Theoretical Plan and Standards for Accuracy

As Korres emphasizes, it is not enough to merely draw geometric shapes over the features of a plan reduced to a scale of 1:100.\textsuperscript{46} Instead, proposed geometric shapes must be verified through analytic geometry. In other words, a superimposed drawing should correspond to the elements they overlap not only visually, but also mathematically through Cartesian coordinates with interrelationships expressed algebraically, and with lines described in terms of slopes and curves with coefficient-based formulas, for example. Naturally, such a strict standard places a damper on continued attempts to theorize about ancient architectural plans, but the gains in credibility are arguably well worth the endeavor.

\textsuperscript{45} 14.666 m is only 0.026 m (or 0.18\%) in excess of 14.640 m. 14.640/24 = 0.610 m; 14.640/45 = 0.325 m; 24.4/40 = 0.610 m; 24.4/75 = 0.325 m.

\textsuperscript{46} Korres 1994, p. 80.
Another point of emphasis has been that the degree of accuracy in a plan’s theoretical geometry must approximate the tolerances in the actual construction.47 Determining the accuracy of the built form, however, elicits a bit of circular reasoning, since many of its elements must be measured against the very same theoretical plan that its author attempts to support.48 This need not be the case for every feature, however. In Temple A, for example, there exist slight variances in the noncontracted columnar interaxes of the lateral colonnades, such as 3.050 m and 3.034 m,49 that are likely to relate to a theoretical constant rather than an intentional irregularity.

On a larger scale, we may note that Temple A’s naos (9.272 m wide, including its walls) lies not in the exact center of the stylobate (15.965 m wide), but an imperceptible 0.067 m off axis (Fig. 5).50 Given the general predilection for symmetry even in conjunction with “optical refinements,” one would be hard-pressed to argue the plausibility of this feature as intentional. From the outer wall to the edge of the stylobate, the distances on the western and southern sides of the naos measure 3.380 and 3.375 m, respectively, and the diverging measurement of 3.313 m on the eastern side represents an error of ca. 1.98%.51

Still, it may be inadvisable to isolate this error in the eastern pteron, since the final built form is the product of multiple interrelating components. The most conservative approach would be to calculate the percentage of tolerance according to the entire width of the stylobate. This calculation should pertain to the theoretical plan rather than the actual plan, with the only difference being the addition of the “missing” 0.067 m from the eastern side of the temple, resulting in a width of 16.032 m for the stylobate and an overall width of 18.142 m (see Appendix 1). In order to maintain the strictest possible tolerance in my analysis of this theoretical plan, I will cap the standard for accuracy at 0.42% in accordance with the divergence discussed here.52

It is important to emphasize that this addition to the width in the theoretical plan is slight, and does not in any way “stack the deck” for the results of the analysis that follows. Instead of adding 0.067 m to the narrower side, we may be justified in adjusting for symmetry in the theoretical plan either by maintaining the actual width and shifting the naos to the center (see Appendix 2),53 or by reducing the width of the naos by 0.067 m in order to balance the sides evenly (see Appendix 3). As the calculations provided in Appendixes 2 and 3 demonstrate, the results for each of these alternative theoretical plans remain well under the strict tolerance

47. Korres 1994, p. 79.
48. In Korres’s words (1994, pp. 79–80), theorists “refuse to be bound by the methodological requirement that the degree to which a theoretical definition (whether metrological, geometric, or whatever) approximates to the actual building should be no less than the degree of accuracy with which the building itself was constructed (which in any case such theorists are incapable of conceiving).”
49. Schazmann and Herzog 1932, pl. 2.
50. Schazmann and Herzog 1932, p. 6, pl. 2. For this and all of the following measurements of the naos and pronasos, dimensions relate to the outside plane of the walls rather than the socle.
51. 0.067/3.380 m. For these measurements and others, see Fig. 5, and Schazmann and Herzog 1932, pl. 2. 0.067/16.032 m = 0.42%.
52. This solution would be consistent with the views of Temple A’s excavators, who explain the displacement as the result of an earthquake that shifted the entire cella; see Schazmann and Herzog 1932, p. 6. For the problems with this theory, see n. 54, above.
of 0.42%, and in fact produce results closer to 0% in the case of several dimensions. The rationale for privileging the theoretical plan in Appendix 1, therefore, is not to provide the most convincing analysis, but rather to adjust for symmetry in a way that most thoroughly relates to the measurements of the actual plan; when 0.067 m is added to the eastern side of the stylobate, the eastern and western sides of the plan equal one another as well as the side behind the southern wall of the naos.54

The theoretical plan of $18.142 \times 33.280$ m solves one problem but leaves another unresolved. On the one hand, our expectation for integral proportions in the overall plan is satisfied, since the theoretical plan results in a nearly perfect 6:11 form.55 On the other hand, the rationale for the placement of the naos and pronaos remains unclear. While the distance of the walls of the naos from the edge of the euthynteria maintains an equal 1:1:1 ratio on the sides and rear, the space before the antae of the pronaos shares no integral relationship with these distances.56 Nor may we readily discern any meaningful proportional relationship in the length-to-width dimensions of the naos and pronaos.57 As I argue below, this lack of observable correspondences pertains to a process of design grounded not in arithmetical relationships between orthogonal dimensions, but rather to a geometric procedure executed with the rule and compass. This geometry is quite simple, though it requires some detail and rigor to substantiate it. In the following section, I demonstrate how we may recover the plan's specific design process through analytic geometry.

**Geometric Analysis**

To properly analyze the plan, I rely on simple calculations based on the published measurements of Temple A, with the only adjustment being a centered naos, flanked on either side by equal distances of 0.380 m from the outer walls of the naos to the edges of the stylobate.58 All relevant diagonal relationships in the plan are mathematically verified and expressed in the footnotes with reference to a single quadrant of a two-dimensional coordinate system. In addition, Appendixes 1–3 with accompanying Figure 23 provide magnitudes, equations, and tolerances that demonstrate the proposed geometry according to measurements for all three theoretical plans described in the prior section. Whenever relevant, the location of features will be given as Cartesian coordinates, in which the southeastern corner of the euthynteria's outer edge is at the origin 0, 0, and the extreme northwest

54. The southern side measures $4.430$ m from the exterior face of the wall of the naos to the outer edge of the euthynteria, which is essentially equal to the $4.435$ measurement of the western side (see Fig. 5). Adding $0.067$ m to the narrower eastern side of the stylobate, therefore, produces a nearly 1:1:1 ratio for all three sides.

55. $(18.142/6) \times 11 = 33.260$, a difference of only 0.02 m from the plan's $33.280$ m length.

56. The distance from the pronaos to the stylobate edge, were it preserved on the northern facade, would be $5.742$ m. The distance from the pronaos to the outer edge of the euthynteria is $6.797$ m. Of these two measurements, the closest integral ratio I can find is a 2:3 relationship between the lateral and rear distances to the euthynteria ($4.435$ m) and that of the front ($6.797$ m), with an implausible tolerance of 2.1%.

57. The overall dimensions of the naos and pronaos are $9.272 \times 22.053$ m. Here, the closest integral ratio is 3:7, whose tolerance of 1.9% is again unacceptable.

58. Thus, the distances between the exterior walls of the naos and each long outer edge of the euthynteria equal $4.368$ m, rather than the present $4.368$ and $4.435$ m; see Fig. 5. See also n. 34, above.
Figure 12. Restored theoretical plan of Temple A with geometric underpinning

59. For consistency, all magnitudes are rounded to the millimeter.

60. From a point located on the plan’s long central axis at [9.071, 12.101], a theoretical line to [0, 0] measures 15.123 m, which is the square root of the sum of the squares of 9.071 and 12.101. From the same coordinates on the central axis, a theoretical line to the external corner of either anta measures 15.124 m. Specifically, the external corner of the western anta is at [13.749, 26.483]. Through simple subtraction, we find these coordinates at distances of 4.678 m and 14.382 m from [9.071, 12.101]. The sum of these figures (15.123 m + 15.124 m) finds a theoretical diameter of 30.247 m; see Appendix 1.

61. (30.247 m/5) x 3 = 18.148 m, a difference of only 6 mm from the plan’s overall width of 18.142 m, and therefore a tolerance of less than 0.1%; see Appendix 1.
proportion that should give pause to our skepticism: the distance from the theoretical circumcenter to the plan's southern edge and the overall width of the temple share a 2:3 ratio, again with a tolerance of less than 0.1%. We may illustrate this correspondence with a baseline $x-x'$ of 3 units drawn across the entire width of plan at the ordinate corresponding to the theoretical center point of the circumference, along with a line $y-y'$ of 2 units drawn from the circumcenter to the edge of the euthynteria (Fig. 12).

Geometrically, we may express this relationship through the algorithm of two circumferences with a radius of 2 units, each centered on either terminus of baseline $x-x'$ (Fig. 13). The larger circumference, which is centered at the middle of $x-x'$, intersects with the smaller circumferences exactly at the points of the outer corners of the euthynteria. Both the mathematical proof for and significance of these intersecting points are revealed by the whole-number ratio of the diameters of the smaller and larger circles, equaling 4:5 with a tolerance of less than 0.1%. When conceived in relation to the overall width of the temple (the 3 units of $x-x'$), this final dimension brings the geometric principle underlying the architect’s system into striking clarity: the 3:4:5 dimensions of a

62. (18.142 m/3) $\times$ 2 = 12.095 m, a difference of only 6 mm from the ordinate at 12.101 m, and therefore a tolerance of less than 0.1%.

63. The theoretical diameter of the larger circumference equals 30.247 m (see n. 60, above), which has a ratio of 5:4 with 24.202 m (the diameter corresponding to the radius of 12.101 m in the $y$ dimension from the baseline $x-x'$ to either back corner of the euthynteria at [0, 0] and [18.142, 0]), with an error of less than 0.1% calculated by the difference divided by the magnitude: (30.247 m/5) $\times$ 4 = 24.198 m, a difference of 4 mm from 24.202 m. (24.202 m/4) $\times$ 5 = 30.253 m, a difference of 6 mm from 30.247 m.
In effect, this geometric form ABC lies at the heart of the design, with the compass centered midway along its hypotenuse and the circumference coinciding with its angles and lines (Fig. 14).

We should understand this geometric underpinning and its compass-based construction as interdependent. Even in the Roman period, architects did not work with a square, let alone a T square. Instead, the method of producing perpendicular lines with the highest precision employed a rule and compass, with straight lines drawn through circumferential intersections in the same manner that is revealed through this analysis of Temple A. It has already been observed that Roman buildings such as amphitheaters would commonly begin with a Pythagorean triangle, and arrive at the final design using the compass through various stages. This Roman use of the Pythagorean triangle recalls a conceptually similar manner of

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64. The theoretical diameter of the larger circumference equals 30.247 m (see n. 60, above). The magnitudes of 18.142 m (the plan’s overall width, or baseline x-x’), 24.202 m (the diameter corresponding to the radius of 12.101 m in the y dimension from the baseline x-x’ to either back corner of the euthyn- teria at [0, 0] and [18.142, 0]), and 30.247 m = 3:4:5, with a maximum error in any dimension of less than 0.1%.


constructing Ionic column bases that was familiar to Greek architects as early as the Archaic period.\textsuperscript{67} The transparency of the plan of Temple A may allow us to understand how a Hellenistic architect might construct the Pythagorean triangle itself. The formula appears to consist of a baseline of 6 units, upon the center of which a compass with a radius of 5 units is set, and on the ends of which are set compasses with radii of 4 units. By these means, the intersections could be joined to form the perpendicular lines of the triangle’s sides as well as the diagonal of its hypotenuse (Fig. 15). In the case of Temple A, it appears that the larger circumference of this geometric construct remained in place to define the extent of the pronaos at the antae (Fig. 14).

To dismiss these results would now require us to posit a confluence of three separate coincidences of whole-number proportions (3:4:5) with a maximum error that is consistent with the strictest possible standard of tolerance observable in the actual building, along with a fourth (and more conspicuous) coincidence that these proportions engender an integral geometric form of central significance to Greek mathematics. Moreover, the circumscription of a Pythagorean triangle graphically expresses Thales’ theorem: three perpendicular bisectors meet at a circumcenter located on the hypotenuse, which runs the length of the circle’s diameter (Fig. 16).\textsuperscript{68} In turn, the basic proportions that the Pythagorean triangle yields establish the location of the theoretical center point and the diameters of the circumferences (Fig. 15). In the face of these internal correspondences and their pertinence to Euclidian geometry, the balance concerning this resulting form obviously falls heavily on the side of intentional design rather than chance.

There is yet another integral proportion that completes the geometric underpinning of the temple’s plan. The diagonal across the naos from corner to corner including its external walls shares a 1:1 correspondence with the total width of the temple, with a difference of only 0.1\%.\textsuperscript{69} From a theoretical central point located on the cross-axes of the naos, therefore, the distance to either edge of the temple’s width and each of the external corners of the naos is essentially equal. This congruency suggests a circumferential underpinning to the design of the naos, whose diameter shares a

\begin{figure}[ht]
\centering
\includegraphics[width=0.5\textwidth]{figure15.png}
\caption{Proposed general construction of a 3:4:5 Pythagorean triangle according to circumferential intersections, with dashes indicating the baseline}
\end{figure}

\textsuperscript{67} For the Pythagorean triangle and column bases, see Gruben 1963, pp. 126–129.

\textsuperscript{68} By definition, a Pythagorean triangle is a right triangle; see Euc. \textit{Elem.} 3.31.

\textsuperscript{69} Width and length of naos including its walls equal 9.272 and 15.572 m, respectively. According to the Pythagorean theorem, then, we square each and find the square root of their sum, thus finding 18.123 m. If we take the 0.019 m difference between 18.123 and 18.142 m (the total adjusted width of the temple) and divide by either 18.123 or 18.142 m, we find a difference of 0.1\%. 
whole-number 3:5 ratio with the diameter of the large circle, with a tolerance of 0.1% (Fig. 17). If we accept these circumferences as a guiding method for the placement of features within the plan, their ratio would call to mind Vitruvius’s formula of 3:5 circumferences for the main proportions of plans in peripteral round temples (Vitr. 4.8.2).

One indication that the circumferences here suggest an intentional geometric underpinning is their planar interrelationship. The distance separating the ordinates of their theoretical center points is 0.115 m. In a scale plan, special markings are required to make this separation perceptible (Fig. 18). Before considering why the architect might have centered his compass at different points a hair’s width apart in his design, we might consider this separation in relation to the theoretical proportions and actual dimensions to which it corresponds: the separation of these ordinates

70. \((30.247 \text{ m}/5) \times 3 = 18.148 \text{ m}\), a difference of 0.1% from 18.123 m; \((18.123 \text{ m}/3) \times 5 = 30.205 \text{ m}\), a difference of 0.1% from 30.247 m.

71. The coordinates of the theoretical center point of the larger circle are [9.071, 12.101] (see n. 60, above). For the smaller circle, the coordinates of the center point are [9.071, 12.216]. The ordinate here is determined by the sum of the center point of the naos and the distance of the naos from the south outer edge of the euthynteria: \((15.572 \text{ m}/2) + 4.43 \text{ m}\). For the distance separating the theoretical center points of the larger and smaller circles: \(12.216 - 12.101 = 0.115 \text{ m}\).
represents a 0.38% difference, so we remain within the strictest standard for theoretical tolerances of 0.42% calculated according to the constructional inexactitude found in the actual building.\textsuperscript{72}

On the other hand, the applicability of this standard here is dubious. Although we cannot conclusively determine the precise metrological system underlying Temple A, the fact remains that the architect or builders would have needed to convert any conceptual circumferential geometry to actual measurements for orthogonal distances. After all, we cannot expect masons to have laid out the building according to invisible circles with an eye to maintaining a shared theoretical center point. Due to such necessary adjustments in the planning and building process, it is natural that deviations from original design elements are bound to occur. Since we lack secure access to this intermediary stage of metric specification, the relevance of a precise calculation for the percentage of error in a common center point (such as 0.38%) may be limited. Instead, we may conceive of the divergence in more experiential terms: in a building over 33 m long, we find the two theoretical circumcenters of the integrally proportioned circumferences at points only 0.115 m apart, or less than the length of a small child's hand in relation to the distance from floor to vaults in the cathedral.

\textsuperscript{72} As in the calculation of error relating to the difference of 0.67 m in the widths of the ptera (0.42% in relation to the entire width of the stylobate), the difference is here calculated according to the complete geometry, as represented by the diameter of the larger theoretical circumference: 0.115/30.247 m = 0.38%.
of Notre Dame in Paris. In a structure where even the width of the stylobate is off by 6.7 cm, an additional inexactitude of 4.8 cm for an invisible feature is insignificant, particularly when that feature was no longer relevant during the actual building process.

The Schematic Plan

Leaving aside the security that mathematical justification affords, I will now suggest possible ways in which this geometry relates to the Hellenistic architect's process of designing Temple A. Unlike the theoretical demonstration above, the following analysis takes into account the proportions of the actual building. My intention here is to explore further questions relating to the design process, integrating what I hope is well-grounded speculation with the results of the above geometric analysis.

In designing Temple A, the architect would have needed to harmonize the 3:4:5 triangle underlying the placement of the naos and pronaos with the 6:11 ratio of the overall plan. Keeping in mind how a compass is centered, it is worth emphasizing that the simplest way of working with the tool is to conceptualize circumferences in terms of radii rather than
diameters. In this way, one need not resort to half-number divisors, such as 2.5, in order to create a whole-number diameter such as 5 units. In producing radii of 3, 4, and 5 on a baseline of 6, therefore, the architect would have created a 6:8:10-unit triangle. Extending this same divisor to the overall plan, an additional 3 units in the y dimension produces the final 6:11 ratio of the temple's plan, which repeats the 6 × 11 number of columns for the intended colonnades and simplifies the process of drawing by maintaining integers.

This demonstration of the architect’s method of locating the wall termini according to circumferences still does not explain the rationale behind where they were placed along those circumferences. It is tempting to suggest a simple circumference-based algorithm whereby the architect might have worked out these placements. On the baseline x–x′ of 6 units, center the compass on the termini and center, drawing three circles of equal radius. Repeat this procedure three times, each with radii of 3, 4, and 5 units, finding the location of the walls and corners according to the circumferential intersections (Figs. 19–21). Despite the appeal of the resulting plans, however, it would be inadvisable to adopt this procedure. As Korres recognizes, we cannot draw conclusions concerning geometry on the basis of how that geometry appears to coincide with features when overlaid on a scale plan.73 Rather, we must replicate such results mathematically. Unlike

Figure 20. Restored theoretical plan of Temple A overlaid with intersections of circumferences with radii of 4 units

74. In the case of the 3-unit radii (Fig. 19), it has already been established that the naos corners are set 9.062 m from the cross-axis at [9.071, 12.216]. Because the plan is symmetrical, only one corner of the naos needs to be considered here: from [0, 12.216] to the naos corner at [4.435, 4.430] we find x and y dimensions of 4.435 and 7.786 m to calculate a diagonal distance of 8.961 m, showing a difference of 1.1% from the expected 9.062 m.

In the case of the 4-unit radii (Fig. 20), we may reference the line of either long pronaos wall. That of the western wall intersects with the central circumference at [13.707, 23.279], as given by the distance of 4.636 m from the midpoint of the plan to the external wall of the naos and the 12.101 m radius, resulting in a distance of 11.178 m from the baseline to the intersection in the y dimension (12.101² = 4.636² + y²). The western wall's intersection with the lateral circumference occurs at [13.707, 23.360], as given by the wall's distance of 4.435 m from the outer edge of the euthynteria and the 12.101 m radius, resulting in a distance of 11.259 m from the baseline to the intersection in the y dimension (12.101² = 4.435² + y²). The difference of these intersections of 0.081 m in the y dimension is a tolerance of 0.7%.

Still, even in cases where proposals hold up to such scrutiny, one consideration deserves recognition. There is, of course, a gap between our method of verifying the plan through analytic geometry and the ancient method of converting the location of its features into magnitudes for
the actual building. We might, therefore, ask how an architectural scale drawing would have been created in the Hellenistic period. This question is especially relevant to the planning of Doric temples, where interstitial columnar contraction precluded convenient repetition of uniform paving slabs that ensure conformity to a grid-based plan.

A reasonable answer in the case of Temple A, I suggest, lies in a simple intuitive process that begins with the initial schematic sketch before the completion of the detailed drawing (see Fig. 22): (1) within the smaller circle, set the lines of the exterior walls of the naos at the rear and sides with approximately equal distances to the outside edges of the overall plan in accordance with the principle of symmetry; (2) where the lateral lines again intersect with the circumference of this same circle, set the spur walls separating the naos and pronaos; (3) in conjunction with these same lateral lines, set the antae at the intersection with the circumference of the larger circle. In the drawing process itself, this result is most easily achieved in a way that is similar to what I describe above: first set the locations of the corners and the antae by establishing equal distances from the plan’s edges, and then mark these points with the compass set on the termini and center of the baseline $x-x’$. In these ways, the logic of the overall design maintains symmetry with interrelationships that are circumferential, which is in keeping with a process of drawing that relies upon the rule and compass.

Figure 21. Restored theoretical plan of Temple A overlaid with intersections of circumferences with radii of 5 units
If we again consider the hypothesis of a modular-based metrology, we can speculate on one manner in which the plan's designer might have established scale. Since the placement of features depends upon circumferential considerations, while the production of elements such as paving slabs must be related to orthogonal dimensions, it would appear that the drawing precedes the scaling in the following way. In privileging a fixed magnitude such as 45 Doric feet for the colonnade axes of the front and rear, the architect could measure the remaining elements in the drawing (such as the dimensions and placements of the walls and the varying dimensions of the individual slabs that make up the stylobate and steps) against these established distances and fix their sizes according to scale. By its nature, this procedure would be inexact for two reasons. In the first place, the expectation of symmetry in the final built form would dictate equal values for the distances from the exterior naos walls to the edge of the stylobate at both the sides and rear, when in fact the geometry of the drawn form would show a very slight discrepancy between the lateral and rear distances; indeed, the separation of 0.38% in the centers of the theoretical circumferences (Fig. 18) is likely to be a result of this very consideration. 

Secondly, the plan's designer would need to measure the features on the drawing surface by hand and convert them to varying values. Unlike the case with Ionic temples, the varied spacing of columns in a Doric temple such as that at Kos dictated that individual slabs could not repeat an established prototype. Distances, therefore, would need to be subdivided into varying units for the paving slabs in accordance with the spatial contractions.

In the end, therefore, the measurements would have needed to address the individual paving slabs in addition to the overall size of the stylobate or euthynteria. Because of the multiple steps in this process, and the slight modifications bound to occur in each of these steps, it is not reasonable for us to theorize intended values for each element and dimension of the plan, given as measurements down to the dactyl. Instead, the significant result of this study remains the revealed correspondence of the overall form to a rational, theoretical geometry in which the percentages of error remain within the strictest possible tolerance found in the actual construction.
CONCLUSIONS

A metrological analysis of Temple A in the Asklepieion at Kos suggests that the triglyph module theory proposed by Wilson Jones for 5th-century Doric temples may be applicable to this Hellenistic example. This theory cannot, however, account for the locations of features not associated with the temple’s trabeation, such as the walls of the naos and pronaos. Since Temple A was created in an era when the kind of drawn plan described by Vitruvius is likely to have been already commonplace in Ionic temples, we are justified in asking how its plan might address the considerations of design particular to the Doric order, where transparent orthogonal relationships established with a grid were not possible. A geometric analysis that responds to the methodological issues addressed by Korres demonstrates that a circumscribed Pythagorean triangle forms the basis of Temple A’s design, in which circumferences determine the placements of the plan’s principal features. Unlike the more difficult problem of verifying the modular theory, the evidence for this geometric system rests solely upon the internal, measurable correspondences that conform to Euclidian norms. Furthermore, we can replicate these results both by calculation and with CAD software. In combination with the modular theory, we can speculate that the colonnade axes may have played a role in establishing scale in the drawn plan, by allowing for the conversion of relative dimensions into actual values for the building. The full implications of the results of this analysis cannot be explored in the present study, but a few observations merit brief comment.76

In its details, the design process proposed here runs counter to the simple grid approach used in Ionic temples, as well as differing from current ideas about the way in which Doric temples were designed. Wilson Jones insists on the principle of “facade-driven” design for Doric temples, in contrast to the “plan-driven” design for Ionic temples.77 In other words, architects designed Doric temples strictly according to the commensuration of elements in the facade, as opposed to the creation of a guiding plan that determined the layouts of Ionic temples. Yet given the mixing of the architectural orders as early as the 5th century B.C.—most famously witnessed in the Parthenon—we might question such categorical notions of mutual exclusiveness, particularly in buildings as late as the Hellenistic period. As discussed above, it appears that the triglyph module may very well have played a significant role in the design of Temple A’s facade. One might wonder, however, why ancient architects who are likely to have been trained in the details of both orders should necessarily have repressed planning tendencies solely due to the employment of a particular module. After all, Ingrid Rowland has convincingly demonstrated the very notion of mutually exclusive “orders” to be an early modern transformation of Vitruvius’s genera, which, like ancient buildings themselves, accommodate notable degrees of interchangeability.78 That Vitruvius should omit a discussion of taxis in relation to Doric temples probably reflects his bias toward the traditions of Ionic design that formed the core of his architectural training.79 If Vitruvius’s ignorance of Doric taxis stemmed from this limited

76. In an article currently in progress, I assess the results of the present analysis along with other considerations in the larger context of ancient Greek architectural drawing, masonry tools, and methods of planning.


78. The notion of the “orders” as rigidly defined categories appears to begin with Renaissance thinkers in the milieu of Raphael and Bramante, continuing later with Serlio, Palladio, and Vignola; see Rowland 1994; Howe and Rowland 1999, p. 15.

background, there is no reason why we should perpetuate his ignorance by extending it retrospectively to Hellenistic architects and their buildings.

With support from the results of this analysis, it is even worth speculating on the special potential of the Doric order for a higher degree of sophistication in the drawing-board design process. At least in the case of Temple A, the variations in columnar placements in a Doric temple might have motivated an alternative approach to the location of the internal features within the plan.\(^8\) What appears to have resulted was a system more interesting than the simple arithmetical relationships characteristic of the grid plan, but also one that was perhaps too innovative for reuse and continued development. Perhaps partly for this reason, and partly because of the “decline” in the production of Doric temples altogether, the possibly Doric-related method found in Temple A may have disappeared from common practice well before Vitruvius picked up his pen. Yet Temple A was not the final instance of this approach, which appears to have extended even beyond the Doric order and into a Hellenistic–Roman context, where temple plans continued to demonstrate the application of the Pythagorean triangle and 3:5 circumferences as their guiding geometry.\(^9\) Ultimately, however, the geometry of form characteristic of Temple A might have its most recognizable legacy not in the cryptomethodic \textit{taxis} of the architect’s drawing board, but in the shapes that Roman \textit{opus caementicum} finally allowed for permanent expression in three dimensions. Framed in this way, the fully experiential intersection of the idea and its reflection would give rise to a new aesthetic that would have been unimaginable in the Hellenistic architectural theory of Vitruvius.

\(^8\) In Temple A, the lateral corner column interaxes measure ca. 2.7 m, as opposed to the other average interaxes of ca. 3.05 m. In the facade and rear colonnades, the corner column interaxes measure ca. 2.7 m, while the second and central interaxes measure 3.065 and 3.080 m, respectively; see Schazmann and Herzog 1932, pl. 2.

\(^9\) These geometric approaches are found in two of the earliest hellenizing temples in Italy during the Republican period: the Temple of Juno at Gabii of ca. 160 B.C. and the round temple of ca. 120–100 B.C. in Rome’s Forum Boarium. For the geometry of the Temple of Juno at Gabii, see Almagro-Gorbea 1982; Jiménez 1982, esp. pp. 63–74; Almagro-Gorbea and Jiménez 1982; Coarelli 1987, pp. 11–21. For the round temple, see Rakob and Heilmeyer 1973. An elaborated analysis of such geometry, its significance, and the connections between these examples and the work at Kos discussed in the present study are themes that I explore in a follow-up article (in progress) focused on Roman architecture.
APPENDIX 1
THEORETICAL PLAN A

Select locations, coordinates, magnitudes, and equations for theoretical plan A are given below. The coordinates correspond to measurements in meters taken by Schazmann and Herzog (see Fig. 5),[82] converted here to an 18.142 x 33.280 quadrant with origin 0, 0 and limit 18.142, 33.280 at the southeastern and northwestern extremes, respectively (Fig. 23). For additional equations, see text and notes above.

<table>
<thead>
<tr>
<th>Location</th>
<th>Relevant Circumference</th>
<th>Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0, 0</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>18.142, 0</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>24.202</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>4.435, 4.430</td>
</tr>
<tr>
<td>E</td>
<td>2</td>
<td>13.707, 4.430</td>
</tr>
<tr>
<td>F</td>
<td>2</td>
<td>13.707, 20.002</td>
</tr>
<tr>
<td>G</td>
<td>2</td>
<td>4.435, 20.002</td>
</tr>
<tr>
<td>H</td>
<td>1</td>
<td>13.749, 26.483</td>
</tr>
<tr>
<td>I</td>
<td>1</td>
<td>4.393, 26.483</td>
</tr>
</tbody>
</table>

Circumference 1
(Circumcenter 9.071, 12.101)

Definitions
AC = Diameter = Radius 1 + Radius 2
Radius 1 = distance from circumcenter to A (or B)
Radius 2 = distance from circumcenter to H (or I)

Magnitudes
X, Y distances from circumcenter to A
X1: 9.071 - 0 = 9.071
Y1: 12.101 - 0 = 12.101

X, Y distances from circumcenter to H
X2: 13.749 - 9.071 = 4.678

82. Schazmann and Herzog 1932, pl. 2.
Figure 23. Restored theoretical plan of Temple A with geometric underpinning and indicated locations corresponding to Cartesian coordinates.

Equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius 1² = X₁² + Y₁²</td>
<td>9.071² + 12.101²</td>
</tr>
<tr>
<td>Radius 1 = 15.123</td>
<td></td>
</tr>
<tr>
<td>Radius 2² = X₂² + Y₂²</td>
<td>4.678² + 14.382²</td>
</tr>
<tr>
<td>Radius 2 = 15.124</td>
<td></td>
</tr>
<tr>
<td>AC = 15.123 + 15.124 = 30.247</td>
<td></td>
</tr>
</tbody>
</table>

Differences in Magnitudes

<table>
<thead>
<tr>
<th>Difference</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.247 - 30.205</td>
<td>0.042</td>
</tr>
<tr>
<td>18.148 - 18.123</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Tolerances

<table>
<thead>
<tr>
<th>Tolerance</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.042 / 30.247</td>
<td>0.1%</td>
</tr>
<tr>
<td>0.025 / 18.123</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

Pythagorean Triangle Equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>hypotenuse² = AB² + (Y₁ × 2)²</td>
<td>18.142² + 24.202²</td>
</tr>
<tr>
<td>hypotenuse = 30.247</td>
<td></td>
</tr>
</tbody>
</table>

Difference in Magnitude

<table>
<thead>
<tr>
<th>Difference</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC - hypotenuse</td>
<td>30.247 - 30.247 = 0</td>
</tr>
</tbody>
</table>

Tolerance

<table>
<thead>
<tr>
<th>Tolerance</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 / 30.247</td>
<td>0%</td>
</tr>
</tbody>
</table>
Select locations, coordinates, magnitudes, and equations for theoretical plan B are given below. The coordinates correspond to measurements in meters taken by Schazmann and Herzog (see Fig. 5), converted here to an 18.075 x 33.280 quadrant with origin 0, 0 and limit 18.075, 33.280 at the southeastern and northwestern extremes, respectively, and a symmetrically centered naos (see Fig. 23, scaled for the slightly differing dimensions and coordinates of Appendix 1).

<table>
<thead>
<tr>
<th>Location</th>
<th>Relevant Circumference</th>
<th>Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
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<td>0, 0</td>
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<tr>
<td>B</td>
<td>1</td>
<td>18.075, 0</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>18.075, 24.224</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>4.402, 4.430</td>
</tr>
<tr>
<td>E</td>
<td>2</td>
<td>13.674, 4.430</td>
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<tr>
<td>F</td>
<td>2</td>
<td>13.674, 20.002</td>
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<tr>
<td>G</td>
<td>2</td>
<td>4.402, 20.002</td>
</tr>
<tr>
<td>H</td>
<td>1</td>
<td>13.716, 26.483</td>
</tr>
<tr>
<td>I</td>
<td>1</td>
<td>4.360, 26.483</td>
</tr>
</tbody>
</table>

_Circumference 1_
(Circumcenter 9.038, 12.112)

**Definitions**

- AC = Diameter = Radius 1 + Radius 2
- Radius 1 = distance from circumcenter to A (or B)
- Radius 2 = distance from circumcenter to H (or I)

**Magnitudes**

- X, Y distances from circumcenter to A
  - X1: 9.038 - 0 = 9.038
  - Y1: 12.112 - 0 = 12.112
- X, Y distances from circumcenter to H

---

83. Schazmann and Herzog 1932, pl. 2.
Equations
Radius 1^2 = X1^2 + Y1^2
Radius 1^2 = 9.038^2 + 12.112^2
Radius 1 = 15.112
Radius 2^2 = X2^2 + Y2^2
Radius 2^2 = 4.678^2 + 14.371^2
Radius 2 = 15.113
AC = 15.112 + 15.113 = 30.225

Circumference 2
(Circumcenter 9.038, 12.216)
Diameter^2 = (DE)^2 + (EF)^2
Diameter^2 = 9.272^2 + 15.572^2
Diameter = 18.123

6:10 Ratio of Circumferences 1 and 2
Equations
(18.123 / 6) \times 10 = 30.205
(30.225 / 10) \times 6 = 18.135

Differences in Magnitudes
30.225 – 30.205 = 0.020
18.135 – 18.123 = 0.012

Tolerances
0.020 / 30.225 < 0.1%
0.012 / 18.123 < 0.1%

Pythagorean Triangle
Equations
hypotenuse^2 = AB^2 + (Y1 \times 2)^2
hypotenuse^2 = 18.075^2 + 24.224^2
hypotenuse = 30.224

Difference in Magnitude
AC – hypotenuse
30.225 – 30.224 = 0.001

Tolerance
0.001 / 30.224 < 0.1%
Select locations, coordinates, magnitudes, and equations for theoretical plan C are given below. The coordinates correspond to measurements in meters taken by Schazmann and Herzog (see Fig. 5),\textsuperscript{84} converted here to an $18.075 \times 33.280$ quadrant with origin 0, 0 and limit 18.075, 33.280 at the southeastern and northwestern extremes, respectively, with the width of the naos reduced .067 m in order to provide symmetry (see Fig. 23, scaled for the slightly differing dimensions and coordinates of Appendix 1).

<table>
<thead>
<tr>
<th>Location</th>
<th>Relevant Circumference</th>
<th>Coordinates</th>
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<td>A</td>
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<tr>
<td>B</td>
<td>1</td>
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<td>C</td>
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<td>18.075, 24.224</td>
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<td>D</td>
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<td>H</td>
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<td>13.682, 26.483</td>
</tr>
<tr>
<td>I</td>
<td>1</td>
<td>4.393, 26.483</td>
</tr>
</tbody>
</table>

\textit{Circumference 1}  
(Circumcenter 9.038, 12.107)

\textit{Definitions}
AC = Diameter = Radius 1 + Radius 2  
Radius 1 = distance from circumcenter to A (or B)  
Radius 2 = distance from circumcenter to H (or I)

\textit{Magnitudes}
X, Y distances from circumcenter to A  
X1: $9.038 - 0 = 9.038$  
Y1: $12.107 - 0 = 12.107$
X, Y distances from circumcenter to H  
X2: $13.682 - 9.038 = 4.644$  
Y2: $26.483 - 12.107 = 14.376$

\textsuperscript{84} Schazmann and Herzog 1932, pl. 2.
Equations
Radius 1² = X₁² + Y₁²
Radius 1² = 9.038² + 12.107²
Radius 1 = 15.108

Radius 2² = X₂² + Y₂²
Radius 2² = 4.644² + 14.376²
Radius 2 = 15.108

AC = 15.108 + 15.108 = 30.216

Circumference 2
(Circumcenter 9.038, 12.216)
Diameter² = (DE)² + (EF)²
Diameter² = 9.205² + 15.572²
Diameter = 18.089

6:10 Ratio of Circumferences 1 and 2
Equations
(18.089 / 6) × 10 = 30.148
(30.216 / 10) × 6 = 18.130

Differences in Magnitudes
30.216 - 30.148 = 0.068
18.130 - 18.089 = 0.041

Tolerances
0.068 / 30.216 = 0.2%
0.041 / 18.089 = 0.2%

Pythagorean Triangle Equations
hypotenuse² = AB² + (Y₁ × 2)²
hypotenuse² = 18.075² + 24.214²
hypotenuse = 30.216

Difference in Magnitude
AC - hypotenuse
30.216 - 30.216 = 0

Tolerance
0 / 30.216 = 0%
REFERENCES


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